Jake Chandler

Department of Philosophy, University of Glasgow,
67-69 Oakfield Avenue, Glasgow G12 8QQ
✉ J.Chandler@philosophy.arts.gla.ac.uk

UNIVERSITY of GLASGOW

[1] Admin, Overview & Propositions and Sets

0. Outline

1. Admin
2. Course overview
3. Propositions and sets
4. Set theory basics
5. Operations on sets

1. Admin

- Course composition:
  - 16 x 1h lectures (possibly a couple more)
  - 4 x 1h seminars
- Time & location: Caird room, Tuesdays & Fridays 3-4pm.
- Course website:
  - Location: Philosophy Moodle http://moodle2.gla.ac.uk/philosophy/moodle/
  - Contents: syllabus, reading list, lecture slides, downloadable articles.
  - Lecture slides posted online 1-2 days after each lecture

1. Admin

- Readings:
  - 1 compulsory piece + optional readings per lecture
  - Feel free to contact me for even further reading if interested
- Assessment:
  - 1 examination
  - 1 formal essay, due in towards end of term (exact date TBA)
    - Essay topic to be posted on Moodle in the next few weeks.
    - For further details on essay policies (style, length, submission guidelines, etc.), see Honours Handbook, available on departmental website.

1. Admin

- Finally…
  - Office hours: Dept of Philosophy, ground floor, room 307 on Mondays & Wednesdays 10-11am.
  - Make use of them!

2. Course overview

- Aim of the course:

  (i) To provide an introduction to some basic concepts commonly found in the more 'technical' literature on rational belief, be it in epistemology proper, in philosophy of science or in philosophy of social science (e.g. phil. of economics).

  (ii) To equip the student with some important formal skills (basic set theory and probability theory) that complement those acquired in the symbolic logic modules.
- Regarding (i), more specifically, we will be primarily looking at the merits and weaknesses of a *hugely* influential view sometimes known as 'Bayesianism'.

2. Course overview

- As *I* will use the term (there is some dispute here), Bayesianism is committed to three core claims, one descriptive and two normative.
- *Descriptive claim*: Just like loving and wanting, believing comes in *degrees*. In other words, we have degrees of belief (aka 'credences') in various things.
- More formally: for any agent *S* and time *t*, there exists a function $\text{Bel}_{S,t}$ that maps members of a set of objects of belief (things that can be believed) onto *S*'s degrees of belief in those things at *t*.
- This is fairly uncontroversial and is meant to be reflected in the following kinds of assertions:

  'I don't really / kind of / strongly believe that *P*.' or 'I am not very /pretty / absolutely confident / sure / certain that *P*.'

2. Course overview

- Note: many Bayesians have little to say about what degrees of beliefs *are* - i.e. they don't offer a philosophical analysis thereof.
- However, they *do* commonly make claims about various, possibly contingent, properties thereof (more on this later).
- *Normative claim (1)*: At any one given time, the degrees of belief of a rational subject are '*probabilistically coherent*', i.e. more precisely, if *S* is rational at time *t*, $\text{Bel}_{S,t}$ is a probability function (more on this later).
- *Normative claim (2)*: Over time, as evidence is acquired, rationality requires that the degrees of belief of a subject be updated by a procedure known as '*conditionalisation*' (more on this later).

2. Course overview

- In the course of this module we will also notably be looking at:
  - attempts to analyse the notion of *evidential support* within the Bayesian framework (aka 'Bayesian confirmation theory),
  - the connection, if any, between (i) the *graded* conception of belief discussed in the bulk of this course (believing that *P* to degree *d*) and (ii) an equally common *binary* conception of belief (believing that *P* full stop; reflected in assertions such as: 'I believe what you said').
- *Warning*: as mentioned in the syllabus on the Moodle, whilst this course isn't a mathematical course as such, there *is* a certain formal component (though nothing too scary).
- A very basic grasp of propositional logic and some simple college algebra (manipulating inequalities, etc) would be helpful.

3. Propositions and sets

- In what follows, I will - fairly uncontroversially - assume:

  The objects of belief are *propositions*.
- I shall also follow standard practice in the formal literature on belief and - more controversially - endorse:

  SET: the proposition that *X* = the set of possible worlds in which *X*. (Note: sets of possible worlds are also commonly known as 'events' in mathematics)

  E.g.: the proposition that Benazir Bhutto was assassinated = the set of possible worlds in which Benazir Bhutto was assassinated
- *Note (1)*: there is considerable amount of metaphysical controversy over what a possible world is supposed to be (assuming, as I will, that such a kind of thing exists in the first place).

3. Propositions and sets

- Possible worlds have been taken to be, amongst other things:
  - Mereological sums of concrete objects (e.g. Lewis).
  - Properties (e.g. Stalnaker)
  - Etc.

  (See Melia [2003] for an introductory survey.)
- We'll (thankfully) steer clear of these issues in what follows.
- *Note (2)*: SET is a philosophically controversial thesis, for a number of reasons.
- On this view, for instance:
  - *Sets can have truth values.* This does seem awkward.

3. Propositions and sets

E.g.: compare 'The proposition that Bhutto was assassinated is true.' (fine) and 'The set of all possible worlds in which Bhutto was assassinated is true.' (not-so-fine).

- *Propositions are fairly coarsely individuated.* According to SET, propositions that are true in exactly the same possible worlds count as the same propositions. Many find this counterintuitive.

  E.g.: according to SET, the proposition that *X* is trilateral = the proposition that *X*'s internal angles sum to 180°, or again, the proposition that Superman is Clark Kent = the proposition that Superman is Superman.

  (Note: it follows that believing that *X* is trilateral to degree *d* = believing that *X*'s internal angles sum to 180° to degree *d*)

3. Propositions and sets

- I will have to leave these issues aside for the remainder of the course.
- Now for a quick refresher on (extremely basic) set theory, most of which should be familiar.

4. Set theory basics

- *The basic idea*:
  - A *set* is simply a collection of items (objects, properties, sets,…), known as the *elements* or *members* of the set.
  - A set can (surprisingly) contain no elements; such a set is known as the *empty set*.
  - The set of all relevant items in a given conversational context is known as the *universal set*.
- *Some notation*:
  - $A, B, C,\ldots$ for sets; $a, b, c,\ldots$ for elements.
  - '$a \in B$' stands for 'a is a member of B'
  - '$a \notin B$' stands for 'a *isn't* a member of B'
  - Ω denotes the universal set; $\varnothing$ denotes the empty set.

4. Set theory basics

- *Specifying sets*: two popular methods.
  - *List notation*: list each element in the set or, if the set is countably infinite, list the first few elements.

    E.g.: the set of positive even integers = $\{2,4,6,8,10,\ldots\}$.
  - *Predicate notation*: specify a predicate whose extension corresponds to the elements of the set.

    E.g.: the set of positive even integers = $\{x \mid x$ is a positive even integer$\}$.

    Note: it is well know since Russell that the use of certain predicates to specify sets lead to paradoxes (e.g. being a set that isn't a member of itself; the set of all sets satisfying that property is a member of itself iff it isn't a member of itself).

4. Set theory basics

- *Identity:* $A = B$ iff, for every x, $x \in A \leftrightarrow x \in B$.
- Note: it follows from conditions on set identity that there is only one empty set and one universal set.
- *Subsets*:
  - A is a *subset* of B ($A \subseteq B$) iff, for all x, $x \in A \rightarrow x \in B$
  - A is a *proper subset* of B ($A \subset B$) iff $A \subseteq B$ and $\neg(B \subseteq A)$
  - Like $\in$, both $\subseteq$ and $\subset$ can be negated with a slash (e.g. $\not\subset$)
  - Identity can also be captured in terms of subset relations: $A = B$ iff $A \subseteq B$ and $B \subseteq A$.
  - Important: don't confuse the membership relation with the subset relation!

4. Set theory basics

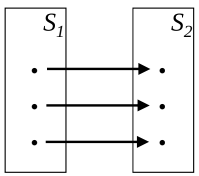
- Note that, according to SET:
  - When 'Ω' denotes the set of all possible worlds, it denotes the necessarily true proposition (true in all possible worlds).
  - 'Ø' denotes the necessarily false proposition (false in all possible worlds).
  - If '*P*' denotes a set of possible worlds $P \neq \Omega \neq \emptyset$, it denotes a contingent proposition.
  - When *P*, *Q* denote sets of possible worlds, $P \subseteq Q$ is a notational variant for *P*'s entailing *Q* (i.e. there being no world in which *P* is true but *Q* false).
  - When *P*, *Q* denote sets of possible worlds $P = Q$ is a notational variant for *P* and *Q* being equivalent.

4. Set theory basics

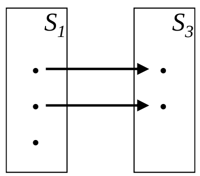
- *Cardinality*:
  - The number of elements in a set *S* is denoted $|S|$
  - If $|S| = 1$, *S* is a *singleton* set
  - If $|S| = 0$, *S* is the empty set (Ø)
  - $|S_1| = |S_2|$ iff S_1 and S_2 can be put into *one-to-one correspondence* (i.e. iff the members of S_1 and S_2 can be paired up such that each member of S_1 is paired up with exactly one member of S_2 and vice versa)

4. Set theory basics

- S_1 and S_2 can be put into 1-to-1 correspondence:



- S_1 and S_3 *cannot* be put into 1-to-1 correspondence:



- $|S_1| > |S_2|$ iff (i) a proper subset of S_1 can be put into one-to-one correspondence with S_2 but (ii) a proper subset of S_2 *cannot* be put into one-to-one correspondence with S_1.

4. Set theory basics

- Note that sets can have either *finite* or *infinite cardinality*.
- Surprisingly enough perhaps, infinite cardinalities come in different sizes (proof omitted)…
- A set *S* is *countable* iff its cardinality is either (a) finite or (b) infinite and equal to the cardinality of the set of natural numbers $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ (in this second case we say that the set is *denumerable*).
- In other words, *S* is countable iff a subset, proper or not, of $\mathbb{N}$ can be put into 1-to-1 correspondence with *S*.
- A set is *uncountable* iff its cardinality is strictly greater than the cardinality of $\mathbb{N}$.

Reference

- Melia, J. [2003] *Modality*. Chesham: Acumen. Ch5 'Extreme Realism' & Ch6 'Quiet Moderate Realism'

Next lecture: 'More on Sets & Intro to Probability Calculus'

- Reading:
  - Weisberg, J. [unpublished]: 'A Probability Primer for Philosophers', excluding section 3 on conditional probability.
- Note: I have also posted the following short chapters on the Moodle, from which my set theory 101 has been drawn (there is some extra content, which you can ignore).
  - Partee, B et al [1990]: *Mathematical Methods in Linguistics*. Dordrecht: Kluwer. Chs 1-2.