Jake Chandler

Department of Philosophy, University of Glasgow,
67-69 Oakfield Avenue, Glasgow G12 8QQ
✉ J.Chandler@philosophy.arts.gla.ac.uk

UNIVERSITY of GLASGOW

[11] Indifference (ctd.)

0. Outline

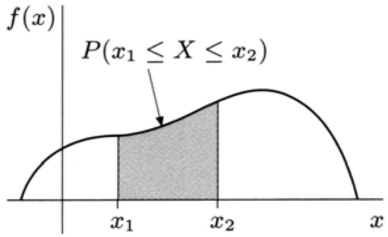
1. The Principle of Indifference (ctd.)

2. Responding to the paradoxes

1. The Principle of Indifference (ctd.)

- Recap on the definitions from last time:
  - Informally, a *random variable X* is a quantity that can take on different values in different possible worlds, i.e. a function from possible worlds to a set of numbers.
  - A *continuous* random variable is a random variable that can take on *uncountably* many values (e.g. any value in some non-degenerate interval of the reals).
  - Where *X* is a continuous r.v., a *probability density function* is (roughly) a function *f* such that the probability that $x_1 \leq X \leq x_2$ is given by the area under the graph of *f* between x_1 and x_2.
  - Formally: *f* is such that $\Pr(x_1 \leq X \leq x_2) = \int_{x_1}^{x_2} f(x)dx$, the integral of *f* on $[x_1, x_2]$)

1. The Principle of Indifference (ctd.)



- Ok, back to **PI**.
- Counterexample #2: van Fraassen's [1989] *cube factory* case.

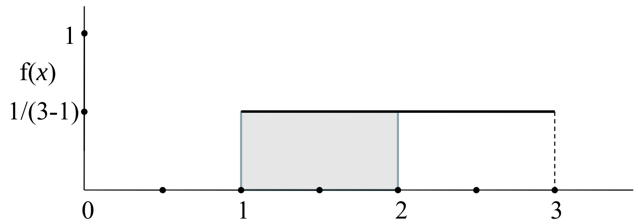
1. The Principle of Indifference (ctd.)

- *S* knows that a certain factory produces cubes with edge length between 1 and 3 but has no further relevant information regarding edge length.
- Question: what should the value of $Bel_S(1 \leq L \leq 2)$ be (where *L* is a random variable corresponding to the edge length of the next cube off the production line)?
- *S*'s epistemic state is such that, for any two subintervals of [1, 3] $I = [x_1, x_2]$ and $I^* = [x_1^*, x_2^*]$ such that $x_2 - x_1 = x_2^* - x_1^*$, *S* isn't justified in preferring to believe $L \in I$ over $L \in I^*$.

  E.g.: *S* isn't justified in preferring to believe $1 \leq L \leq 2$ over $2 \leq L \leq 3$, etc.

1. The Principle of Indifference (ctd.)

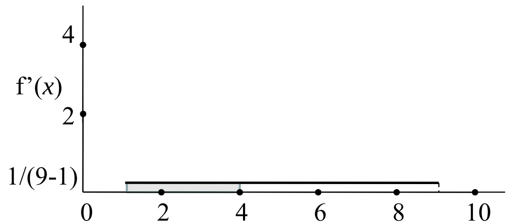
- By **PI**, *S*'s state of belief regarding the value of *L* should be representable by a '*uniform*' pdf over the interval [1, 3]:



- Note: the uniform distribution function over [*a*,*b*] is given by $f(x) = 1 / (b-a)$ and $\int_{x_1}^{x_2} 1/(b-a)dx = (x_2 - x_1)/(b-a)$
- So we have $Bel_S(1 \leq L \leq 2) = (2 - 1) / (3 - 1) = 0.5$

1. The Principle of Indifference (ctd.)

- But now *S* also knows that cubes produced have a side area *A* between 0 and 4 but has no further relevant information wrt *A*.
- So again, by **PI**, *S*'s state of belief regarding the value of *A* should be representable by a uniform pdf over the interval [1, 9].



- So we have $Bel_S(1 \leq A \leq 4) = (4 - 1) / (9 - 1) = 0.375$

1. The Principle of Indifference (ctd.)

- But $1 \leq L \leq 2$ iff $1 \leq A \leq 4$!
- **PI** again yields contradictory prescriptions.
- Suggestion: there is a privileged r.v. with respect to which **PI** should be applied (Gillies [2004:46])
- This may seem at least conceivable in the cube factory case: we have an *asymmetry* between the different r.v.s.
- *L* isn't related to *A* in the way that *A* is related to *L*: the transformation needed to get from the former to the latter (i.e. $f(x) = x^2$) is different from the transformation needed to get from the latter to the former (i.e. $f'(x) = \sqrt{x}$).
- One might therefore be tempted to find some argument to the effect that one r.v. is more 'natural' to use than another.

1. The Principle of Indifference (ctd.)

- Counterexample #3: von Mises' *wine/water* case.
- *S* knows that she has a 10cc mixture of wine and water of which at least 1cc is wine and at least 1cc is water.
- Question: what should *S*'s d.o.b. in there being more water than wine be?
- Call the ratio of wine to water *X*.
- There is more water than wine iff $1/9 \le X \le 1$.
- By **PI**, *S*'s state of belief regarding the value of *X* should be representable by a uniform pdf over the interval [1/9, 9].
- So we have $\text{Bel}_S(1/9 \le X \le 1) = [1-(1/9)] / [9-(1/9)] = (8/9) / (80/9) = 0.1$

1. The Principle of Indifference (ctd.)

- Now call the ratio of water to wine *Y*.
- There is more water than wine iff $1 \le Y \le 9$.
- Again, by **PI**, *S*'s state of belief regarding the value of *Y* should be representable by a uniform pdf over the interval [1/9, 9].
- So we have $\text{Bel}_S(1 \le Y \le 9) = (9-1) / [9-(1/9)] = 72 / 80 = 0.9$
- But of course $1/9 \le X \le 1$ iff $1 \le Y \le 9$: contradictory prescriptions again.
- Now notice that $X = 1/Y$ and $Y = 1/X$: this time, the relationship is *perfectly symmetric*.
- Unlike the cube case, no asymmetry can be exploited to argue in favour of privileging one r.v. over the other.

1. The Principle of Indifference (ctd.)

- One last paradox, in a somewhat different vein…
- Counterexample #4: the *two envelope* paradox.
- Background:
  - Bayesians have something to say not only about how we should regulate our degrees of belief, but how we should make choices on the assumption that our degrees of belief are so regulated ('Bayesian decision theory'; *hugely* influential).
  - Definition: the *expectation* of a random variable, in the discrete case, is the probability-weighted sum of its possible values.
  - E.g.: the expected number of offspring *X* of an animal is given by $E(X) = Pr(X=0)\times 0 + Pr(X=1)\times 1 + Pr(X=2)\times 2 + \ldots$

1. The Principle of Indifference (ctd.)

- In addition to belief functions, agents are taken to have 'desire functions' (aka *utility* functions; $U_S(\bullet)$) that map propositions onto their valuations of these propositions.
- A popular maxim:

  MaxEU: An agent should make choices that maximise his expected utility.
- Formally:

  MaxEU: given a partition $P = \{P_1, P_2, \ldots\}$ of Ω and a set of mutually exclusive and collectively exhaustive acts $A = \{A_1, A_2, \ldots\}$, a rational agent *S* will choose the act A_j such that for all $i \neq j$:

  $$\sum_k Bel_S(P_k | A_j) \times U_S(P_k \cap A_j) > \sum_k Bel_S(P_k | A_i) \times U_S(P_k \cap A_i)$$

1. The Principle of Indifference (ctd.)

- E.g.:
  - I am trying to decide whether to bring white or red wine to the party tonight.
  - If they serve up beef, then white would be a disaster (U = -1), although red would be perfect (U = 1).
  - If they serve up chicken, then red wouldn't be fantastic (U = 0) but white would be perfect (U = 1).
  - Conditional on my bringing red, I am 75% confident that beef will be served and 25% confident that chicken will be served (my hosts will try to not embarrass me)
  - Conditional on my bringing white, my degrees of belief are reversed.

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- Decision table (conditional d.o.b.s in brackets):

	Beef	*Chicken*
Red	1 (0.75)	0 (0.25)
White	-1 (0.25)	1 (0.75)

- Expected utilities:
  - Red: $1 \times 0.75 + 0 \times 0.25 = 0.75$
  - White: $-1 \times 0.25 + 1 \times 0.75 = 0.5$
- **MaxEU** recommends me to bring a bottle of red.

1. The Principle of Indifference (ctd.)

- So… the paradox:
  - S is offered a choice between two envelopes: E_1 and E_2.
  - He is told that one of the envelopes contains twice as much money as the other one does.
  - After he has chosen one of them, and before he opens it, he is offered the opportunity to switch to the other.
  - Ok, say he chooses E_1.
  - We now offer him to switch to E_2. Should he?
  - Call X the amount of cash in E_1, whatever it may be.
  - Now either E_2 has half that amount ($1/2X$), or it has twice that amount ($2X$).

1. The Principle of Indifference (ctd.)

- Now S's epistemic position doesn't favour either option over the other.
- So by **PI**, S should have $\text{Bel}_S(E_2 \text{ has } 1/2X) = \text{Bel}_S(E_2 \text{ has } 2X) = 0.5$
- Expected utilities of switching vs not:
  - Don't switch: $\text{Bel}_S(E_1 \text{ has } X) \times X = X$
  - Switch: $\text{Bel}_S(E_2 \text{ has } 1/2X) \times 1/2X + \text{Bel}_S(E_2 \text{ has } 2X) \times 2X = 1¼X$.
- By **MaxEU**, S should switch.
- Say he does.
- We now offer him the chance to switch back to E_2. Should he?

1. The Principle of Indifference (ctd.)

  - Well… call Y the amount of cash in E_2, whatever it may be.
  - Now either E_1 has half that amount ($1/2Y$), or it has twice that amount ($2Y$).
  - …
  - Now we have *S* forever switching and never ending up getting any money. Clearly not a rational prescription!
- There are two obvious potential guilty parties here: **PI** and **MaxEU**.
- Most decision theorists would be unhappy to give up on **MaxEU**.
- So **PI** has to go.
- For more on this, see for e.g. Jackson et all [1994] (plenty of refs on google).

Reference

- Jeffreys, H. [1939]: *Theory of Probability*. Oxford: OUP.
- Jaynes, E.T. [1968]: 'Prior Probabilities', *IEEE Transactions On Systems and Cybernetics* 4(3):227–241.
- Jackson, F., P. Menzies & G. Oppy [1994] 'The two envelope paradox', *Analysis* 54(1): 43-45.
- van Fraassen, B. [1989]: *Laws and Symmetry*. Oxford: OUP. Ch 12 'Indifference: The Symmetries of Probability'.

Next lecture: 'Updating Belief'

- Reading:
  - Howson, C. & P. Urbach [1993]: *Scientific Reasoning: the Bayesian approach, 2nd Edition*. LaSalle: Open Court. Chapter 6 'Updating Belief'.
- Further reading:
  - Joyce, J. [2004] 'Bayesianism', in Mele & Rawling (eds.) *The Oxford Handbook of Rationality*. Oxford: OUP. Section 4 'The Bayesian Theory of Learning' (very brief)
  - Weisberg, J. [ms] 'Varieties of Bayesianism' sections 3.3 - 3.5