**Jake Chandler**

Department of Philosophy, University of Glasgow,
67-69 Oakfield Avenue, Glasgow G12 8QQ
✉ J.Chandler@philosophy.arts.gla.ac.uk

UNIVERSITY of GLASGOW

# [15] Confirmation (ctd.)



## 0. Outline

**1. Confirmation theory: non-Bayesian approaches (ctd.)**

**2. Confirmation theory: the Bayesian line**



## 1. Confirmation theory: non-Bayesian approaches (ctd.)

- *Possible response #3*: **EQC** is false.
- According to Hempel's account of confirmation, $E = \neg R(a)$ & $\neg B(a)$ confirms not only

  $H_1 = (\forall x)\ (R(x) \supset B(x))$,

  but also

  $H_2 = (\forall x)\ (R(x) \supset \neg B(x))$.

  *Proof*: Let $H_3 = (\forall x)\ (\neg R(x))$. $\mathrm{Dev}_{I(E)}(H_3) = \neg R(a)$ and hence $E \vDash \mathrm{Dev}_{I(E)}(H_3)$, so $E$ directly Hempel confirms and hence, according to Hempel's account, confirms $H_3$. Now, $H_3 \vDash H_2$, so by **SCC**, which is true according to Hempel's account, $E$ confirms $H_2$. ■



## 1. Confirmation theory: non-Bayesian approaches (ctd.)

- Scheffler [1963] finds this particular aspect of Hempel's account counterintuitive.
- His suggestion: $E$ confirms a universal statement $(\forall x)\ (F(x) \supset G(x))$ iff it '*favours*' it over its 'contrary' $(\forall x)\ (F(x) \supset \neg G(x))$.
- He then wants to use this insight, in the Ravens argument, to hang on to intuitive (ii) but ditch counterintuitive (v) (thus denying **ECQ**).
- Indeed, whilst $(\forall x)\ (R(x) \supset B(x))$ and $(\forall x)\ (\neg B(x) \supset \neg R(x))$ are logically equivalent, they have logically *non*-equivalent contraries, namely, respectively:
  - $(\forall x)\ (R(x) \supset \neg B(x))$ (i.e. $(\forall x)\ (\neg R(x) \vee \neg B(x))$)
  - $(\forall x)\ (\neg B(x) \supset R(x))$ (i.e. $(\forall x)\ (B(x) \vee R(x))$)

## 1. Confirmation theory: non-Bayesian approaches (ctd.)

- So the idea is to find a formal account of 'favouring' that yields the result that $\neg R(a)$ & $\neg B(a)$:
  - *favours* $(\forall x)(\neg B(x) \supset \neg R(x))$ over $(\forall x)(\neg B(x) \supset R(x))$
  - *doesn't favour* $(\forall x)(R(x) \supset B(x))$ over $(\forall x)(R(x) \supset \neg B(x))$.
- Scheffler defines the notion of favouring a universal statement over its contrary in Hempelian terms, as follows:

  **Favouring**: $E$ favours $(\forall x)(F(x) \supset G(x))$ over $(\forall x)(F(x) \supset \neg G(x))$ iff, on Hempel's account, $E$ would have been be said to confirm the former and disconfirm the latter.

  (According to Hempel**,** $E$ confirms $H$ iff there is a set of propositions $S$ such that (i) $S \vDash H$, (ii) $E$ directly H.-confirms every member of $S$; $E$ disconfirms $H$ iff $E$ confirms $\neg H$.)



## 1. Confirmation theory: non-Bayesian approaches (ctd.)

- And this, it turns out, does just the job.
- Have a think:
  - $\neg R(a)$ & $\neg B(a)$ favours $(\forall x)(\neg B(x) \supset \neg R(x))$ over $(\forall x)(\neg B(x) \supset R(x))$
  - $\neg R(a)$ & $\neg B(a)$ doesn't favour $(\forall x)(R(x) \supset B(x))$ over $(\forall x)(R(x) \supset \neg B(x))$
- This seems fairly neat to me, although we would need an account of why **EQC** seems so plausible when it is in fact false.
- For more on these issues, see Grandy [1967].



## 2. Confirmation theory: the Bayesian line

- The story so far:
  - 2 accounts of confirmation: HD account & Hempelian account.
  - The HD account faces the irrelevant conjunction/disjunction problem.
  - The Hempelian account seems to run into trouble with the ravens, although Scheffler's view seems promising.
- One shortcoming of *both* accounts: (i) they make no provision for the notion of *degree* of evidential support and (ii) it isn't clear how they could be extended to do so.
- But surely we need such a notion! (e.g. 'He was convicted on the basis of exceptionally strong evidence.')



## 2. Confirmation theory: the Bayesian line

- The Bayesian suggestion comes in two stages: a *qualitative* account of confirmation and a *quantitative* account.
- I'll give you the former first (there are a number of ways of wording this; this is the one I find most plausible).
- Where $\mathrm{Bel}_{S,K}$ is the belief function that any agent $S$ rationally ought to have given background knowledge $K$:

  **BayesQual**: $E$ confirms $H$ wrt $K$ iff $\mathrm{Bel}_{S,K}(H|E) > \mathrm{Bel}_{S,K}(H)$. $E$ disconfirms $H$ for $S$ at $t$ iff $\mathrm{Bel}_{S,K}(H|E) < \mathrm{Bel}_{S,K}(H)$.
- E.g.: if various items of knowledge $K$ (e.g. facts about physical symmetry, etc.) would rationally require any agent $S$ (i) to have, for each toss, equal d.o.b of 1/6 in $T_n = i$ $(1 \leq i \leq 6)$, (ii) to assume independence of tosses, it follows that $T_1 = 6$ confirms $T_1 = 6$ & $T_2 = 6$ wrt $K$.

## 2. Confirmation theory: the Bayesian line

- Important note:
  - *Many* Bayesians (aka 'subjectivist Bayesians') hold the view that some *K's rationally underdetermine* degrees of belief.
  - If this is the case, the above proposal doesn't pan out.
  - Alternative proposal: evidential support is *subjective*. *E* confirms *H* relative to belief function $\text{Bel}_{S,t}$.
  - But this seems counterintuitive (subjectivists do bite the bullet though; see Howson [1991:550] on objectivism about support being 'an old habit of thought that dies hard')…
- Note that, according to Bayesians, $\text{Bel}_{S,t,K}$ is a probability function and hence *E* confirms *H* for *S* at *t* iff *E* and *H* are probabilistically *correlated* ($E \not\perp\!\!\!\perp^{+} H$) under $\text{Bel}_{S,t,K}$.



## 2. Confirmation theory: the Bayesian line

- From this, we know that the inequality in **BayesQual** has the following equivalent formulations (see L5):
  - $\text{Bel}_{S,t,K}(E \cap H) > \text{Bel}_{S,t,K}(E)\, \text{Bel}_{S,t,K}(H)$
  - $\text{Bel}_{S,t,K}(E|H) > \text{Bel}_{S,t,K}(E)$
  - $\text{Bel}_{S,t,K}(E|H) > \text{Bel}_{S,t,K}(E|\overline{H})$
  - $\text{Bel}_{S,t,K}(H|E) > \text{Bel}_{S,t,K}(H|\overline{E})$
- We have also already seen some properties the relation of probabilistic correlation (again, see L5). It is:
  - *Symmetric* (hence if *E* confirms *H*, then *H* confirms *E*)
  - *Non-transitive* (hence it isn't generally the case that if *E* confirms $H_1$ and $H_1$ confirms $H_2$, then *E* confirms $H_2$)



## 2. Confirmation theory: the Bayesian line

- Do you reckon that these are intuitive consequences? Can you think of, say, any counterexamples to transitivity?
- Exercise: what do the HD and Hempelian accounts have to say on the matter?
- Some interesting further properties of **BayesQual**…
- **EC** comes out *true*, subject to minor qualifications, the following being a theorem of probability theory:

  **[T14]** Provided $\Pr(E) > 0$ and $\Pr(H) < 1$, if $E \vDash H$, then $\Pr(H|E) > \Pr(H)$.

  *Proof*: in exercise set 2, we proved that, provided $\Pr(E) > 0$ if $E \vDash H$, then $\Pr(H|E) = 1$. ■



## 2. Confirmation theory: the Bayesian line

- **EQC** comes out *true*, again subject to minor qualifications, the following being a theorem of probability theory:

  **[T15]** If $H_1$ and $H_2$ are logically equivalent, $\Pr(H_1) = \Pr(H_2)$ and $\Pr(H_1|E) = \Pr(H_2|E)$

  *Proof*: from two applications of the sentential version of **[T6]**. ■
- **CCC**, however, comes out *false*, the following *not* being a theorem of probability theory (see appendix for counterexample):

  If $\Pr(H_1|E) > \Pr(H_1)$ and $H_2 \vDash H_1$, then $\Pr(H_2|E) > \Pr(H_2)$. (I assume that the conditional probabilities are well-defined)
- **SCC** also comes out *false*, the following *not* being a theorem of probability theory (see appendix for counterexample):

  If $\Pr(H_1|E) > \Pr(H_1)$ and $H_1 \vDash H_2$, then $\Pr(H_2|E) > \Pr(H_2)$.

## 2. Confirmation theory: the Bayesian line

- **BayesQual** agrees with **HD confirmation** insofar as it judges the the following true:

  Provided that $0 < \Pr(H) < 1$ and $0 < \Pr(E) <1$, if $H \vDash E$, then $E$ confirms $H$. (for a quick proof, see Earman [1992:64])
- Now it *isn't* a consequence of the Bayesian view that for any $E$, $H$ and $H^*$, if $E$ confirms $H$, then $E$ confirms $H\&H^*$. (see appendix for counterexample)
- However, the Tacking problem reappears is a slightly weaker form, as the following is a theorem of probability theory:

  **[T16]** For any $E$, $H$ and $H^*$, if $H \vDash E$ then, provided that $0 < \Pr(E) < 1$, it follows that $\Pr(H|E) > \Pr(H)$ and $\Pr(H\&H^*|E) > \Pr(H\&H^*)$. (I'll leave the proof as an exercise)



## 2. Confirmation theory: the Bayesian line

- So, just as was the case with **HD confirmation**, according to **BayesQual**, the following statement (counterintuitively) comes out true:

  'My observation that you have at least three red cards is evidence that you have a handful of red cards and a pocketful of pennies'.
- The account also faces an analogous weaker relative of the irrelevant disjunction problem.
- Finally, **NC** comes out *false* in the general case: it isn't a theorem of probability that

  $\Pr[(\forall x)(F(x) \supset G(x)) \mid F(a)\&G(a)] > \Pr[(\forall x)(F(x) \supset G(x))]$

  (Similar comments apply to **PRI**)



## 2. Confirmation theory: the Bayesian line

- Illustration (from Good [1967]):
  - $S$ know that there are either 100 black ravens and 1 000 000 other birds ($H$) or 1 000 black ravens, 1 white raven, and 1 000 000 other birds ($\neg H$).
  - A bird $a$ is selected at random and found to be a black raven ($B(a)$ & $R(a)$).
  - We have $Bel_{S,K}(R(a)\&B(a)|H) = 100/1\,000\,000 < Bel_{S,K}(R(a)\&B(a)|\neg H) = 1\,000 / 1\,000\,001$: Bayesian disconfirmation!

    (remember that $Bel_{S,K}(H|E) > Bel_{S,K}(H)$ iff $Bel_{S,t,K}(E|H) > Bel_{S,t,K}(E|\overline{H})$)



## 2. Confirmation theory: the Bayesian line

- Another example: Three people, $a$, $b$ and $c$, are in a room. $H$ = 'everyone leaves the room with someone else's hat'. $E$ = '$a$ leaves with $b$'s hat and $b$ leaves with $a$'s hat'.
- Scorecard:

| | EC | SCC | EQC | CCC | PRI | NC | Irrel. Conj | Irrel. Disj |
|---|---|---|---|---|---|---|---|---|
| **HD conf.** | No | No | Yes | Yes | No | No | Yes | Yes |
| **Hempel conf.** | Yes | Yes | Yes | No | Yes | Yes | No | No |
| **BayesQual** | Yes[1] | No | Yes | No | No | No | Sort of | Sort of |

[1] Provided $Bel_{S,K}(E) > 0$ and $Bel_{S,K}(H) < 1$

## 2. Confirmation theory: the Bayesian line

- Regarding the provision of a account of degree of confirmation, the idea is to find a measure of the *extent to which* $Bel_{S,t,K}(H|E)$ exceeds $Bel_{S,t,K}(H)$ (call it $c_{S,t,K}(H,E)$; I'll henceforth omit the subscripts).
- We want such a measure to be such that, at the very least:
  - $c(H,E) > 0$ iff $Bel_{S,t,K}(H|E) > Bel_{S,t,K}(H)$
  - $c(H,E) = 0$ iff $Bel_{S,t,K}(H|E) = Bel_{S,t,K}(H)$
  - $c(H,E) < 0$ iff $Bel_{S,t,K}(H|E) < Bel_{S,t,K}(H)$
- It turns out that there are *many* measures on the market that satisfy this requirement but aren't even *ordinally equivalent*, i.e. they rank pairs of propositions differently in terms of the degree of confirmation obtaining between them.



## 2. Confirmation theory: the Bayesian line

- The most popular (see Fitelson [1999] for more):

$$d(H,E) = Bel(H|E) - Bel(H)$$

$$s(H,E) = Bel(H|E) - Bel(H|\overline{E})$$

$$r(H,E) = \log\left[\frac{Bel(H|E)}{Bel(H)}\right]$$

$$l(H,E) = \log\left[\frac{Bel(E|H)}{Bel(E|\overline{H})}\right]$$

- This leads to many version of **BayesQual**'s quantitative counterpart, the general schema for which is:

  **BayesQual$_x$**: the degree to which *E* confirms *H* for *S* at *t* is equal to $x(H,E)$ (plug in you favourite measure)



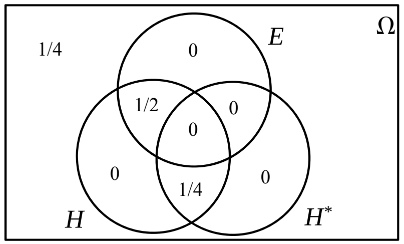
## 2. Confirmation theory: the Bayesian line

- Now Bayesians are often not *only* interested in degree of confirmation for its own sake.
- They also tend to appeal to the notion in trying to resolve various confirmation-theoretic puzzles.
- General strategy:
  - We have some specific case in which **QualBayes** yields the verdict that *E* confirms *H*, when intuitions yield the verdict that it doesn't.
  - Let's claim that although our intuitions are *false*, they are *understandable* as in these cases, the *degree* of confirmation involved is vanishingly small.



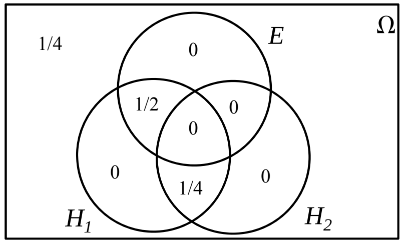
## Appendix

- Proof that it isn't a consequence of the Bayesian view that for any *E*, *H* and *H**, if *E* confirms *H*, then *E* confirms *H&H** (i.e. that it isn't the case that if $Pr(H|E) > Pr(H)$, then $Pr(H\&H^*|E) > Pr(H\&H^*)$).



## Appendix

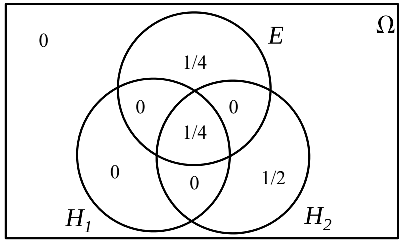
- Proof that it is false that, for all $E$, $H_1$ and $H_2$, if $\Pr(H_1|E) > \Pr(H_1)$ and $H_2 \vDash H_1$, then $\Pr(H_2|E) > \Pr(H_2)$ (pretty much the same model as the previous one):






## Appendix

- Proof that it is false that, for all $E$, $H_1$ and $H_2$, if $\Pr(H_1|E) > \Pr(H_1)$ and $H_1 \vDash H_2$, then $\Pr(H_2|E) > \Pr(H_2)$:






## Reference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## Next lecture: 'Confirmation (ctd.)'

- No set reading.