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[3] Chance (ctd)

2. 'Too improbable given chance alone'

- In the previous lecture, I mentioned the following common line of thought:

  We observe that *O*. Pr(*O* | *CH*) is very low. Hence *CH* must be rejected. (where *O* = there currently exist organisms having the properties that they do and *CH* = these organisms were the product of chance alone)

- In spite of its huge popularity, however, the 'Pr(*O* | *CH*) = low hence reject *CH*' argument doesn't work out.
- The reason for this is simple:

  We aren't licensed to reject hypotheses on the sole basis of their having low likelihoods.

- Of course, we might be *tempted* into thinking that we are…

2. ‘Too improbable given chance alone’

- Indeed, it is very natural to view high conditional probability as an approximation of deductive entailment, a kind of ‘partial entailment’ with the limiting case being $\Pr(Q \mid P) = 1$ if $P \models Q$.
- And the following *is* after all true:

  When a hypothesis *H* deductively entails the falsity of the observation report *E*, it follows that $\Pr(H \mid E) = 0$, as long as $\Pr(E) \neq 0$ (and hence that $\Pr(H \mid E) < \Pr(H_i \mid E)$ for at least one competing hypothesis H_i).

  Proof: if $H \models \neg E$, then *E&H* is a contradiction and by [T2] $\Pr(E\&H) = 0$, hence $\Pr(H \mid E)$, which, by [P4], is equal to $\Pr(E\&H)/\Pr(E)$ is equal to 0 (so long as the ratio is well defined and $\Pr(E) \neq 0$).

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- So perhaps one could say that:

  When a hypothesis *H* confers a probability ≈1 to the falsity of the observation report *E*, it follows that $\Pr(H \mid E) \approx 0$, as long as $\Pr(E) \neq 0$ (and hence, as long as there aren’t too many competing hypotheses, $\Pr(H \mid E) < \Pr(H_i \mid E)$ for at least one competing hypothesis H_i).
- *Not so!!*

  When a hypothesis merely confers a high probability on the falsity of the observation report (i.e. $\Pr(\neg E \mid H)$ = high but < 1), *absolutely nothing whatsoever follows* with respect to the value of $\Pr(H \mid E)$.

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- To see why:

$$\Pr(H|E) = \frac{\Pr(E \,\&\, H)}{\Pr(E)} = \frac{\Pr(H) - \Pr(\neg E \,\&\, H)}{\Pr(E)}$$

$$= \frac{\Pr(H) - \Pr(H)\Pr(\neg E|H)}{\Pr(E)} = \frac{\Pr(H)[1 - \Pr(\neg E|H)]}{\Pr(E)}$$

- So even if $\Pr(\neg E \mid H)$ = high, a low enough value for $\Pr(E)$ and a high enough value for $\Pr(H)$ can leave us with a high value for $\Pr(H \mid E)$
- This doesn't mean that $\Pr(O \mid CH)$ being low isn't bad news for *CH*: other things being equal it *is* bad news. However, it isn't in itself *bad enough* news to reject *CH* outright.

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- To reject the chance hypothesis, we need to consider *CH* in relation to its *competitors*, in terms of both relative prior probabilities and relative likelihoods.
- But there may be a difficulty issue lurking here…
- For instance: whilst we could conceivably arrive at a sensible value for $\Pr(O \mid CH)$, which value should we pick for $\Pr(CH)$?

  (note: I am assuming that we are talking about some kind of objective probabilities here – see Humphreys article)
- In cases like our mammography case, our assessments of the prior probability of the hypotheses are generally themselves based on statistical data: we have epistemic reasons to assign the prior probabilities that we do.

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- But in the case of *CH* and its alternatives, we might not have much by way of epistemic guidance.
- This is in fact a common problem in science (aka *The Problem of the Priors*): whilst values for likelihoods are generally epistemically well-motivated, sensible values for priors sometimes seem difficult to come by.
- An example (from Sober [2004]):
  - Given suitable auxiliary conditions, Newton's Law of Gravitation predicted the return of Haley's comet (i.e. $Pr(HAL|AUX \& NEW) = 1$)
  - But what prior probability should we have assigned to *AUX & NEW*?

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- I do not want to argue that this kind of epistemic situation *is* the case for *CH* and its alternatives, but assuming it *were* the case, where would it leave us?

2. 'Too improbable given chance alone'

- One popular suggestion would be to apply a principle known as the *Principle of Indifference*:

  PI: if we have no prior epistemic reason to think that the elements of a set of *n* mutually exclusive and collectively exhaustive possibilities under consideration $H_1, \ldots, H_n$ don't have equal prior probabilities, then we should assign them equal prior probabilities of 1/*n*.

- This does sound appealing: if we have no prior reason to think that a dice is loaded, it seems legitimate to assume it is fair (i.e. Pr(*1*) = Pr(*2*) = ... = Pr(*6*) = 1/6)
- There may be some mileage in this, but note that PI is contentious: it is thought to lead to inconsistent probability assignments…

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- Example:
  - We know the following: an urn contains white and coloured balls in unknown proportions.
  - We have no prior reason to think that the elements of the following set of mutually exclusive and collectively exhaustive possibilities don't have equal probabilities:

    {*W*, *C*} (where *W* = next ball drawn will be white, and *C* = next ball drawn will be coloured)

  - Therefore, by PI, we should set Pr(*W*) = Pr(*C*) = 1/2
  - But now consider: the set of coloured balls is *itself* made up of red and blue balls in unknown proportions.

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- So it is *also* the case that we have no prior reason to think that that the elements of the *following* set of mutually exclusive and collectively exhaustive possibilities is more likely to be true than the other:

  $\{W, R, B\}$ (where W = next ball drawn will be white, R = next ball drawn will be red, and B = next ball drawn will be blue).
- Therefore, by PI, we should have $\Pr(W) = \Pr(R) = \Pr(B) = 1/3$.
- So by PI, we should both have $\Pr(W) = 1/2$ and $\Pr(W) = 1/3$: contradiction.

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- This case isn't too difficult to solve. Suggestion:

  Require that the possibilities used for the probability assignments in PI form the *finest-grained partition* of the space of outcomes under consideration (i.e. consist only in outcomes that are indivisible into further outcomes).
- In this case PI would then recommend considering $\{W, R, B\}$ rather than $\{W, C\}$ (because C is divisible into R and B).
- This seems ok in this case.
- But there are some more recalcitrant scenarios (e.g. Von Mises' 'Wine/Water Paradox').
- On these, and possible responses thereto, see Gillies [2000: 37-49].

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- Setting these worries aside and assuming that PI is a legitimate recommendation, what are the consequences for our problem?
- The upshot of PI is straightforward. It can be easily proven that:

  In the absence of prior epistemic reasons to assign different probabilities to *CH* and its competitors, according to PI, for any competitor H_i, $\Pr(CH \mid O) > \Pr(H_i \mid O)$ iff $\Pr(O \mid CH) > \Pr(O \mid H_i)$

- In other words: to reject/endorse *CH* (using the principle COMP, discussed in the previous session) we would just need to look at the relative likelihoods of competing hypotheses.

Next lecture: 'Evolution by Natural Selection'

- Reading:
  - Sober, E. [2005]: *Philosophy of Biology*. Oxford: OUP.
    - Ch 1 'What is Evolutionary Theory?', skipping sections 1.4, 1.6 and 1.7,
    - Ch 3 'Fitness', skipping section 3.7.
- Supplementary reading:
  - Lewens, T. [2007] *Darwin*. London: Routledge. Chapter 1 'Life' (for a brief bio of Darwin)

Reference

- Gillies, D. [2000]: *Philosophical Theories of Probability*. London: Routledge.
- Sober, E. [2004]: 'The Design Argument', in W. Mann (ed.) *The Blackwell Guide to Philosophy of Religion*, Oxford: Blackwell.