ELEMENTS OF DEDUCTIVE LOGIC

10. Gaps & Gluts: some semantics (ctd.)

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The supervaluationist logic SV

- In L9:
  - A general view of semantics, suitably for both bivalent and multivalent logics
  - Two logics for reasoning with gaps:
    - Semantics for a logic B3, which seemed ok for presupposition failure, but not ok for future contingents or borderline cases.
    - Semantics for a logic K3, which wasn't ok for presupposition failure, but seemed to do better for future contingents and borderline cases.
    - Some issues for K3: $\nvDash_{K3} \varphi \vee \sim \varphi$ (certainly no good for future contingents)
- Here:
  - A rather interesting alternative to K3...
  - Two logics, LP and RM3, for reasoning with gluts

The supervaluationist logic SV

- As before: $V = \{0, i, 1\}$ and $D = \{1\}$.
- How about truth tables?
- You're in for a surprise...

  The connectives *are not truth-functional*!
- The truth value of a complex sentence is determined by the truth value of its subsentences *only when* these have classical values.
- So what about the general case?!?
- Let v be an assignment of values from $V = \{1, i, 0\}$ to the *atomic* sentences in $\mathcal{L}_S$.

The supervaluationist logic SV (ctd.)

- A resolution v^+ of v is classical valuation that

  (i) agrees with v on its assignment of classical values but
  (ii) substitutes a classical value for every non-classical value assigned by v.

Resolutions

Let the set of atomic sentences be $\{p, q, r\}$, $v(p) = 1$ and $v(q) = v(r) = i$.
The valuation v^+, such that $v^+(p) = 1$, $v^+(q) = 1$ and $v^+(r) = 0$ is a resolution of v.

The supervaluationist logic SV (ctd.)

- We now extend v to more complex sentences, as follows:
  $v(\varphi) = 1$ iff $v^+(\varphi) = 1$ for all resolutions v^+ of v
  $v(\varphi) = 0$ iff $v^+(\varphi) = 0$ for all resolutions v^+ of v
  $v(\varphi) = i$ in the remaining case

Complex sentences in SV

Let $v(\varphi) = i$. What is (i) $v(\varphi \vee \sim \varphi)$ and (ii) $v(\varphi \vee \varphi)$?
For some resolutions v^+ of v: $v^+(\varphi) = 1$ and $v^+(\sim \varphi) = 0$
For the remaining others: $v^+(\varphi) = 0$ and $v^+(\sim \varphi) = 1$
Upshot:

- In both cases, $v^+(\varphi \vee \sim \varphi) = 1$, so $v(\varphi \vee \sim \varphi) = 1$
- However, in the first case $v^+(\varphi \vee \varphi) = 1$, whilst in the second, $v^+(\varphi \vee \varphi) = 0$, so $v(\varphi \vee \varphi) = i$

Comments on SV

- The previous example demonstrates the failure of truth-functionality for the connectives of SV:
  - In both cases the disjuncts have value i
  - In one case ($\varphi \vee \sim \varphi$), the disjunction has value 1
  - In the other case ($\varphi \vee \varphi$), it has value i

  $\Rightarrow$ The value of the inputs isn't enough to determine the value of the outputs!
- In section on K3: some troubling invalidities (e.g. $\nvDash_{K3} \varphi \vee \sim \varphi$).
- What about here?
- It turns out that:

  An argument is SV-valid iff it is classically valid.
- For proof, see Priest (2008, p. 134).
- So in particular, $\vDash_{SV} \varphi \vee \sim \varphi$

Introduction

- In L8: some possible cases of truth value gluts.
  - 'Liar'-style sentences
  - Moral dilemmas
- Let's look at the semantics for two logics to handle these:
  - LP (Priest 1979),
  - RM3.

Priest's 'logic of paradox' LP

- Formally: exact same logic as K3, with the (important!) difference that $D = \{i, 1\}$.
- Interpretational difference: i to be read as 'both true and false'.
- The K3 truth tables make sense on this interpretation too:

$f_\sim$	
1	0
0	1
i	i

$f_\vee$	1	0	i
1	1	1	1
0	1	0	i
i	1	i	i

- For instance, if φ is false and ψ both true and false, then $\varphi \vee \psi$ is both true and false:
  - $\varphi \vee \psi$ is true because ψ is true.
  - $\varphi \vee \psi$ is also false because ψ is false and φ is false.

Priest's 'logic of paradox' LP

- Changing D to $\{i, 1\}$ has a big impact on validity...
- *Observation #1*: there *are* tautologies in LP, in particular

  $\vDash_{LP} \varphi \vee \sim \varphi$.

  Indeed whatever $v(\varphi)$, $v(\varphi \vee \sim \varphi) = 1$ or i and hence is in D.
- *Observation #2*: modus ponens is now invalid

  $\varphi, \varphi \supset \psi \nvDash_{LP} \psi$.

  Countermodel: $v(\varphi) = i$ and $v(\psi) = 0$. It follows that $v(\varphi \supset \psi) = i$, and hence that the premises have values in D, but the conclusion does not.
- Invalidity of MP is not a very nice result...

RM3

- We can recover modus ponens with a small teak.
- RM3: same as LP, but with a different truth table for $\supset$:

$f_\supset$	1	0	i
1	1	0	i
0	1	1	1
i	1	i	i

LP/K3

$f_\supset$	1	0	i
1	1	0	0
0	1	1	1
i	1	0	i

RM3

- Exercise: prove validity of MP in RM3 using the truth table method.

Next session

- Topic: tableau methods for K3 and LP.
- *Optional* reading: Priest (2008), Ch 8. (Warning: tough going for beginners and covers a fair amount more than we will.)

References

- Priest, G. (1979). 'The Logic of Paradox', *Journal of Philosophical Logic* 8, pp. 219-41.