ELEMENTS OF DEDUCTIVE LOGIC

4. Validity of sentential forms and truth tables

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Most descriptive sentential forms: dictionaries

- In the previous session, I introduced the concept of the most descriptive sentential form of an argument.
- When writing down this form, we establish a one-to-one correspondence between the English atomic sentences and various letters from $\mathcal{L}_S$.
- This correspondence is recorded in something called a dictionary.
- To illustrate: the most descriptive sentential forms of our initial argument examples, with associated dictionaries…

Back to our examples

- The most descriptive sentential form of *Cluedo* and *Keys*:

(1) $p \vee q$
(2) $\sim p$

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(3) q

- Dictionaries:

  p = Professor Plum did it / My car keys are at home
  q = Colonel Mustard did it / My car keys are in the office
- This form is known as the disjunctive syllogism.

Back to our examples (ctd.)

- The most descriptive sentential form of *Still out and about* and *Departure*:

(1) $p \supset q$
(2) $\sim q$

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(3) $\sim p$

- Dictionaries:

  p = She is back / He was planning on leaving
  q = The lights are on / He contacted Sam
- This form is known as modus tollens.

Back to our examples (ctd.)

- Finally, The most descriptive sentential form of *Hiding* and *Bar Rumba*:

(1) $p \vee q$
(2) $p \supset r$
(3) $q \supset s$

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(4) $r \vee s$

- Dictionaries:

p = He stayed in Damascus / It'll be crowded
q = He went to Aleppo / It'll be empty
r = He was arrested / It'll be too hot
s = He was murdered / It'll be boring

- This form is known as the constructive dilemma.

Formal validity

- It turns out that, as suggested during the last session, *any* argument that instantiates one of these three sentential forms is valid.
- This is also true of a number of other common sentential forms.
- We say of such forms that they are valid sentential forms (antonym: *in*valid sentential forms).
- In other words: a form is valid if and only if every argument that instantiates it is valid.
- We say that an argument that instantiates a valid form is formally valid.

Notes on formal validity

- Note #1: an argument may well be formally valid even though it instantiates no valid *sentential* form.
- Indeed, as I said earlier, there are *other* kinds of valid forms.
- We will see later in the course that the following is formally valid, although it has no valid sentential form:

Hooligans

(1) Some football fans are hooligans.
(2) All hooligans are bad-tempered.

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(3) Some football fans are bad-tempered. (From (1) and (2))

Notes on formal validity (ctd.)

- Note #2: an argument may well be valid even though it isn't *formally* valid, i.e. instantiates no valid form at all.
- A case in point:

Bachelor

(1) Ben is a bachelor

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(2) Ben is unmarried.

- Most descriptive sentential form: p, therefore q (invalid; can you tell me why?)
- It turns out that none of its other kinds of form are valid either.

Notes on formal validity (ctd.)

- Note #3: an argument that instantiates a valid sentential form will also instantiate sentential forms that are not valid.
- In the previous session, we saw that *Cluedo*, which does have one valid form (disjunctive syllogism), *also* has the following form:

(1) $p \vee q$
(2) $\sim r$

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(3) s

- This form is not valid. (Can you tell me why?)

Spotting a valid sentential form

- Recognising the instantiation of a valid sentential form is obviously a very useful shortcut when evaluating an argument.
- But how do we know when a sentential argument form is valid in the first place?
- This will take me a little time to explain...

Truth-functionality

- Our connectives are commonly taken to have a very particular feature: truth-functionality.
- A connective is truth-functional if and only if the truth value of its output is entirely determined by (= is a function of) the truth value of its input(s).
- *Not all* connectives are truth-functional!! Many cases in point:
  - 'It is well known that...'
  - 'It could have been the case that...'
  - 'It's plain wrong that...'
  - etc.
- Can anyone give me an example showing that these aren't truth-functional?

Truth tables

- The particular way that a certain connective determines the truth value of its output from the truth value of its input(s) is called its semantics.
- The semantics of a truth-functional connective is summarised in a truth table.
- We'll now review the so-called 'classical' truth tables for our connectives, which assume that any sentence is either true ('1') or false ('0') but not both.

  (Later on in this course: non-classical truth tables)

Conjunctions

- A conjunction is true if and only if both its conjuncts are true. Otherwise it is false.

φ	ψ	$\varphi \& \psi$
1	1	1
1	0	0
0	1	0
0	0	0

- Question: isn't temporal priority of the first conjunct over the second also sometimes required for truth?

  Reply: violating this temporal order can make the sentence *misleading* but not *false*.

Disjunctions

- A disjunction is true if and only if at least one of its disjuncts is true. Otherwise it is false.

φ	ψ	$\varphi \vee \psi$
1	1	1
1	0	1
0	1	1
0	0	0

- Question: but isn't a disjunction sometimes considered false when both disjuncts are true?

  Reply: sure; that's known as exclusive disjunction ($\underline{\vee}$). But $\vee$ represents inclusive disjunction.

  We can define $P \underline{\vee} Q$ as $(P \vee Q) \& \sim (P \& Q)$

Negation

- A negation is true if and only if its negand is false. Otherwise it is false.

φ	$\sim \varphi$
1	0
0	1

- Nothing too controversial here.

Conditionals

- A conditional is false if and only if its antecedent is true but its consequent is false. Otherwise it is true.

φ	ψ	$\varphi \supset \psi$
1	1	1
1	0	0
0	1	1
0	0	1

- The conditions under which a conditional is claimed to be false are uncontroversial.
- The rest of the story, however, *is* controversial. More on this later. Please suspend disbelief for now.

Biconditionals

- A biconditional is true if and only if both its terms have the same truth value. Otherwise it is false.

φ	ψ	$\varphi \equiv \psi$
1	1	1
1	0	0
0	1	0
0	0	1

- With all these tables in hand, we can now construct truth tables for *any* kind of sentence whatsoever.
- More on this next time…

Next session

- Topic: more on truth tables